Mind Mapping Fractions, Decimals and Percents

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Nathan Hale Elementary in Sunnybrook School District 171 was recognized by the state for school improvement on the ISAT test in part due to a 21% increase in fourth grade math scores.

One of the most difficult topics for middle school students is fractions. Students spend long hours on worksheets that have them practicing the rules for addition, subtraction, multiplication, and division of fractions. Yet when time comes for testing teachers are at a loss to explain why their students perform poorly. The same can be said to a lesser extent for the student scores on decimals and percent.

The National Council of Teachers of Mathematics addresses the teaching of these topics in the Standards for Grades 6-8 (Principles and Standards for school Mathematics 2000). The specific goals are:

- That students be able to work flexibly with fractions, decimals and percents.
- Select appropriate methods and tools for computing with fractions, decimals and percents.
- Understand the meaning and effects of arithmetic operations with fractions, decimals and integers.
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil.
- Develop and analyze algorithms for computing with fractions and decimals.
- Develop and use strategies to estimate the results of computation and judge the reasonableness of the results.

Is this what is happening in our middle schools? No! What we see almost without exception is a series of worksheet after worksheet that seem to provide no increase in standardized test scores and high school students that continually answer \( \frac{1}{2} + \frac{1}{4} = \frac{2}{6} \). When you ask any high school teacher if their students can do rational number operations in algebra or geometry the answer is almost always no. What happened to all the time spent on worksheets? Was there any learning? Are we just teaching our students what key strokes to perform on a calculator and hoping they can even remember this? Read the assigned chapter, answer the questions at the end of chapter, sit and take lecture notes, complete the worksheet, and take a test are all listed as this is not teaching in How to be an Effective Teacher, (by Harry Wong, pp. 218-219, 1998). Bloom’s taxonomy arranges the verbs into six related groups:

1. Knowledge,
2. Comprehension,
3. Application,
4. Analysis,
5. Synthesis,

Most worksheets involve answering only level 1 type questions. NCTM standards start at level 2 type questions. No wonder children have trouble remembering or learning fractions. NCTM standards talk about communication and connections. Each is very difficult to ask, evaluate, or teach with traditional worksheets.

The solution for more effective lessons involves both the NCTM standards and being an effective teacher. Elementary
teachers have long known that they must establish connections for their students to understand fractions. Often these connections involve making a drawing or diagram of the concept or problem. The effective teacher is the one who has the students doing the work. The one doing the work is the one doing the learning. Often times middle school teachers end up with a high degree of frustration and are worn out because they are the ones doing the work and their students are not doing the learning.

Middle school teachers must spend more time having their students working on developing concepts and less time on abstract practice. This can be accomplished by mind mapping fractions, decimals and percents as an interrelated activity with a connection to the kinesthetic activity of making the related picture or model. Students must see the marriage of picture, fraction, decimal, and percent. Once the basics are well understood it is easy to progress to more difficult problems.

The mind map consists of 5 separate but related parts. A rectangle to show fractions, a dollar sign to show the value in money, a decimal point to show the decimal value, a triangle to show the percent and a 10 by 10 grid to show the appropriate picture.

The first part: rectangle, $, decimal, %, grid are easy for the students to remember. This forms the first part of the connection. The second part of the connection is the pictorial representation of the fraction, decimal and percent. This connected to money, something that all students see and think about each day provide a strong memory basis for the students. The 10 by 10 grid is the picture method recommended and used by the NCTM in the standards.

It is not necessary to use large or complicated fractions. Only the most common should be used, halves, thirds, fourths, fifths, eighths, and tenths. These provide more than an adequate background for a fundamental understanding of fractions decimals and percent. If the students have a firm understanding of the basics of fractions the rest is easy.

A few examples are shown, put in order so that students can also see the equivalent values:
Use all the fractions to help with the connections. You need to use all the halves, all the thirds, all the fourths, all the fifths, all the eighths, and all the tenths. Only a sample is shown above. It is necessary to show all of these to show the connections.

It is now easy for students to find equivalent fractions. They are the ones with the pictures that have shaded the same. The comparison of order of fractions decimal and percent is also as simple as looking at the picture. Basic fractions are easy to reduce simply by using the illustration. Decimal and percent equivalents to fractions can be very visual. Performing addition and subtraction with and without a common denominator is as easy to do as counting squares. More importantly pictures aid students in making estimates. This is especially important if the students are using a calculator. Is this the right answer? Or have I made a mistake with the decimal or denominator. The use of money, with the picture also reinforces the concept of estimation. Estimation is one of the key goals of the NCTM. Multiplication and division of fractions and decimals makes sense to students in picture and money form. No more ½ of $.50 is 1.00! Large posters of the basics should be placed around the room for easy and constant reference. Once the basic fractions and operations have been mastered most students can easily make the transformation to more complicated problems. Not that the problems are any more complicated only that the numbers are larger. Is this important for your students to know? Do we really add sevenths and eights or multiply $2\frac{3}{7} \times 4\frac{1}{3}$? Does this really have a real life connection? Is it tested on State exams or the ACT test? Use your time to teach what they will need in their real life.

Note the physical mind mapping connections: There is the brain friendly connection involving the pictures and symbols. Fractions are shown with a rectangle so that the fraction bar fits parallel to the top and bottom of the rectangle. The decimal point follows the decimal with the dollar sign. A triangle is used to show percent because the side of the percent sign resembles the side of a fraction. Squares to show pictures are drawn in a $10 \times 10$ format to match money, percent and NCTM standards. (NCTM, p. 215) The visual connections must be the first established for the students to make the connections between fractions, decimals and percent. Other connections are shown in money and picture scale. Yes a few worksheets are necessary for practice and review. Yes a few larger problems are necessary, but this is a question of balance. Now the scale has slipped to more and more problems with less and less understanding of the marriage of the basic concepts that bring fractions, decimals and percent together. Calculators can perform the most difficult computation problems, but students must be able to interpret the correctness of their results and they must also be able to make a connection between the use of the calculator and the basics problems. Fractions, decimals and percent should not be taught as individual activities, but must be taught as different representations of the same processes for students to have complete understanding.

When students see these connections, they can choose the method of problem solving that makes most sense to them as recommended by the NCTM standards. Finding 50% of an unknown number is the same as finding $\frac{1}{2}$ of it or in decimal form multiplying by .50. The picture provides the final check of estimation. Have they shaded in the appropriate amount of the given? Likewise, why does dividing by $\frac{1}{2}$ make the number larger? Try dividing by .50 and see what happens? This method also appeals to a variety of learning styles, most importantly.
the visual and kinesthetic learners’ needs are met.

This all fits with current research that says explaining basic concepts behind math problems improves children’s learning. “New research from Vanderbilt University has found students benefit more from being taught the concepts behind math problems rather than the exact procedures to solve the problems. The finding offers teachers new insights on how best to shape math instruction to have the greatest impact on student learning. The research by Bethany Rittle-Johnson, assistant professor of psychology and human development at Vanderbilt University’s Peabody College and Percival Mathews, a Peabody doctoral candidate, is in press at the Journal of Experimental Child Psychology. Teaching children the basic concepts behind the math problems was more useful than teaching children a procedure for solving the problems—these children gave better explanation and learned more. …This adds to the growing body of research illustrating the importance of teaching children concepts as well as having them practice solving problems.” (Science Daily, April 2009). The shapes, the relationships and the pictures all add to the student’s understanding of the concepts of fractions, decimals and percents.

The internet can also provide a valuable tool for both the practice and the understanding of fractions, decimals and percents. There are many web sites that can provide a wide variety of interactive activities and practice that would not be possible for the classroom teacher to duplicate. This provides the help in understanding that is fun and involved for today’s learners. Working math now will become a fun activity and not a bore. These sites must be carefully chosen to be of interest to the students and sound mathematical content.

Look how these activities fit together with the NCTM standards. Students are working flexibly, they understand the meaning, they can select from the appropriate method or tool, they can analyze the different algorithms, they can develop strategies, and estimate the reasonableness of the results. Always refer back to the posters. Students will need to see the connections many times before it all makes sense to them.

New state and national testing standards relate to Bloom’s taxonomy. Level one type questions, knowledge, are rarely asked. Students can simply crunch it out on a calculator. Comprehension, applications, analysis, synthesis, and evaluation are the types of questions asked. This requires a different approach to the teaching of mathematics than the worksheets of the past. Students must evaluate, develop, compare and contrast different methods of doing the problems in order to fully understand the relationships of fractions, decimals and percents.

Years of teaching experience and research now offer all teachers methods of providing effective instruction. State testing evaluates how successful we are at teaching standards using research based methods. This article provides a start in the directing of successful research based instruction.