How do you rate as a teacher of problem solving?

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Are you an effective teacher of problem solving? Teachers often wonder if their students are really thinking critically and reasoning to solve difficult, demanding problem solving tasks. They wonder if they are doing the right things to make sure that problem solving is happening in their classrooms. During the past three years, I have watched teachers implement problem solving. After observing and researching their problem solving practices, I have developed a quiz that might help you decide if what you think you are doing in your classroom is really making students solve problems.

The Quiz

Please answer the following questions to the best of your ability:
1. Do you know how to choose good problems that make connections between mathematical concepts?

2. How do you handle the stress that students experience during problem solving?

3. Do you know each problem inside and out? Do you analyze each problem to tell what concepts it teaches and where students may get stuck?

4. Do you have a bank of thought-provoking questions prepared to ask to make students do the reasoning?

5. After introducing and discussing the problem solving task, do you hand over the work to student groups to solve the problem?

6. Do you let students determine the constraints of the problem?

7. When students ask you questions, do you ask them questions back to make them think?

8. Do you fall into the trap of explaining how students should attack the problem or giving too many “hints” that guide students too much?

9. When problem solving groups hit a snag, what do you do?

10. Are you prepared for groups that finish early?

11. When the session ends, do you make sure that all problem solving groups have the same correct answer?

12. Do you let students debrief and take the sideline to listen to their ideas?

13. Do you leave problems up in the air to motivate students to explore further?

The Answers

So how did you do? Let’s check. With the help of some educational researchers and the veteran facilitators of problem solving I observed, I propose the following answers:
1. Choosing a problem that makes connections between mathematical
concepts is difficult. A way to make sure that you are doing this is to compare your problems to the Task Analysis Guide found in *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development* (Stein, Smith, Henningsen, & Silver, 2000, p. 16). Problem solving tasks are sorted into four categories: memorization, procedures without connections, procedures with connections, and doing math. Although students need to have practice with problems in all these categories, problem solving needs a higher level of cognitive demand and the problems you chose to use should be classified as procedures with connections or doing math. These tasks demand engagement with concepts that underlie rules and formulae, require complex non-algorithmic thinking, can be represented in multiple ways, and stimulate students to make purposeful connections between mathematical concepts, processes, and relationships (Ball, as cited in Stein, Smith, Henningsen & Silver, 2000). Compare the following sample problems from this text (pp.1-2):

<table>
<thead>
<tr>
<th>Low level task</th>
<th>High level task (Doing Math):</th>
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<tr>
<td>(Procedures without Connections):</td>
<td>The Fencing Task</td>
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<tr>
<td>Martha’s Carpeting Task</td>
<td>Ms. Brown’s class will raise rabbits for their science fair. They have 24 feet of fencing with which to build a rectangular pen.</td>
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<td>Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?</td>
<td>a) If Ms. Brown’s students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?</td>
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<td></td>
<td>b) How long would each of the sides of the pen be if they had only 16 feet of fencing?</td>
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<td></td>
<td>c) How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.</td>
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If you are serious about choosing good problems, the best thing you can do is to read the text above. In addition to helping you understand how to analyze each type of task through examples and cases, it also illustrates how to maintain cognitive demand throughout a lesson.

2. Problem solving can really create stressful situations. Members of groups can get into heated discussions about what strategy to use, what the correct answer is, the validity of each others’ ideas, etc. Sixth grade teacher Brenda’s students were intrigued by problem solving, but were also afraid to take risks and possibly be wrong. One of her largest obstacles was overcoming her personal doubts about letting students struggle. She came to realize that struggling with problem solving resulted in more confident problem solvers. She worked to create an environment that fostered dialogue, struggling, risk-taking, critical thinking and sharing of ideas, not competition.

3. Do you need to know each problem inside and out? I used to think that I shouldn’t know every detail of a problem because I didn’t want to lead students to specific strategies that I might use to solve it. Now I know better. As I review specific problems that I use year after year, I realize that the more I know about the problem, the better I can relate the solving process to other related mathematical topics. An example of this can be illustrated by the Locker Problem and the work of another middle school teacher, Belinda. In the problem, 100 lockers are opened and closed based on a sequence of multiples. Students can make a chart or diagram or work through the problem with manipulatives. As they
do so they often jump to conclusions about the patterns they are recording. They see a pattern of open lockers that increases by two between each closed locker. They can then solve for the number of closed lockers under 100, but still may not realize why they are closed. Too late, Belinda realized that she did not understand each step of the problem and what students may be thinking, so the reasoning and discussion behind why some lockers remain opened and others closed was superficial. She needed to redirect their reasoning by asking questions like, “Why are these particular lockers closed?” “How many people touched the closed lockers?” “What makes the closed lockers different than the open ones?” “Is there a reason why some are touched an even number of times or an odd number of times?” and so on to get at why the answers make sense. Belinda’s students never understood the relationship between the number of factors of each number and if its corresponding locker was open or closed. So don’t be surprised if you need to think and research the problems and mathematical ideas behind them, just be sure you are not leading them to a solution.

4. Yes, you need to prepare questions to help students make the connections that are present in a higher level problem. What better time to prepare your questions, than when you are working through the problem yourself. When you find a problem that really challenges your students, save the questions you ask with your lesson plans. Each year that you use your old standby problems, you will see new ways to make math connections and add more questions to your list. Of course, there are questions that will work for almost any problem, as suggested by the Professional Teaching Standards for Teaching Mathematics (1991) and cited in Mewborn and Huberty’s article, Questioning Your Way to the Standards. Some of them are:

   “Does anyone have the same answer but a different way to explain it?”
   “Can you make a model to show that?” “What would happen if…?”
   “Can you predict the next one?”
   “Can you think of a counterexample?” “What is alike and different about your method of solution and hers?” “What ideas that we have learned before were useful in solving this problem?”(p. 244).

5. Hopefully, you answered ‘no’ to this question. After years of teaching mathematics through problem solving, I came to realize that each student needs to think about the problem before being immersed into a group discussion. The strategy “Think, Pair, Share” comes to mind (Lyman, 1981). After the problem is presented and questions concerning the problem are discussed, students need time to formulate their own ideas and the strategies they might use to solve it. Brooks & Brooks (1999) believe that “immediate responses prevent students from thinking through issues and concepts thoroughly, forcing them… to become spectators as their quicker peers react” (p. 115). Allowing 2 to 5 minutes for individual thinking pays off during the problem solving process because students come armed with ideas to share. One teacher I observed (Carol Aljets, personal communication) not only requires that students think on their own first, but asks students to have ideas
written on paper before they share with a classmate.

6. What are the constraints of a problem? They are the little details of the problem that can skew the results of the possible answers that students get. For example, in a problem involving the strategy “make an organized list” students are asked to find all the possible license plates that can be made with the numbers 1, 2, 3, and the letters A, B, C. In the discussion of the problem, students need to analyze the wording to decide if letters and numbers can be used more than once, and if letters and numbers can be mixed. These limits can be decided by the students and will make a difference on their outcomes. If students do not understand the constraints of the problem, their answers will not converge on an appropriate answer.

7. Instead of guiding students to a solution through explaining how to do the work or even giving innocent “hints,” teachers need to encourage inquiry by asking open-ended, thought-provoking questions (Brooks & Brooks, 1999). One approach is to ask students to elaborate on their initial ideas. Brooks & Brooks believe that “Through elaboration, students often reconceptualize and assess their own errors” (p. 111).

What to say when asked a question is difficult to determine. Laurel, a fifth grade teacher, describes how she handled such a dilemma:

Every group wanted help, a clear direction to the solution, but I remained neutral. When I questioned or commented to a group, the other groups were ‘all ears,’ listening for information that they may find useful. Finally, they realized that I wasn’t going to lead them to an answer, and the class began to develop its own ideas. They stopped talking to me and either sat in pondering silence or actively discussed ideas in their group...

Giving a guiding question and/or questions without giving the answer is hard.

When in doubt about how much to say, Laurel suggests that you ask students to tell you what they have tried so far and show their work. Ask questions like, “Explain your ideas behind using this strategy.” “Did you ask all of your group members this question first?” “Is there another way you can check to make sure you are on the right track?” Students often get frustrated when teachers take this approach, but that is okay. In Laurel’s case, she persisted and students began to think for themselves. The question is “Can you outlast them and their frustration to make this change in problem solving ownership?”

8. Instead of showing students methods to try, ask students to share some of their ideas for how they might do the problem. Brooks and Brooks (1999) state, “When teachers share their ideas and theories before students have an opportunity to develop their own, students’ questioning of their own theories is essentially eliminated” (p. 107). I have observed teachers following this lesson sequence: 1) present the problem, discuss what it means and its constraints, 2) students think individually and formulate a strategy, 3) students share strategies within their group, 4) groups think about each strategy and check out its feasibility, then 5) groups share what strategies they think might work best. This additional
discussion point helps groups who might be heading toward a dead end without giving them too much information. Groups have not yet arrived at a solution, so no answers will be shared. This additional step is especially helpful if the task is very challenging.

9. What to do when groups are stuck, and the problem seems to have no end in sight? Does the teacher take over and work it out? No. It is time for a whole group discussion. I have found through my own experiences and that of other teachers, to ask each group to report on the strategies they have tried, this gives other groups the opportunity to look at the problem from a different angle. After all discussion is exhausted, send groups back to work. Repeat the process if necessary or refer to the answer to number 13.

10. Problem solving is like a maze, each group begins moving through the maze at their own pace, making turns at random. There always seems to be one group that makes all the right turns at the right time, so the teachers I observed prepared an extension to the original problem. One easy way is to change the constraints determined by the students earlier and ask the group to report on how this change affects the answer. Another way is to modify the original problem with more variables or larger numbers to make it more difficult. In any event, you need to be prepared for those fast finishers.

11. When the problem solving session is over depends on you. Does every group need to have the correct answer? Any answer? Do half of the groups need to have the correct answer? Is the time for math over? The teachers I observed generally had a good idea of when to stop the session. They felt that at least half of the groups should have a reasonable answer and be working on the extension to the problem while other groups may have arrived at a conclusion that may be partially correct. In some instances, a group may not be close to an answer, but that is okay. Students are building experiences that will help them with future problems.

12. When the time comes to discuss the problem, possible strategies to solve it, connections between mathematical concepts, and the answer, students should be doing the talking (Brooks & Brooks, 1999). At this point in the process, they are the experts on their own work. Taking away the excitement of sharing their work is a crime. Mary Kay, a fifth grade teacher believes that her “main role is to study my students, assess their progress, and organize sharing time so students view a variety of solutions… to help students build concepts, learn new strategies and reflect on their own learning.” In this vein, hand groups overhead transparencies and markers and ask them to explain every step of the path they took to get to a solution. Should the teacher ever step in? Only as a facilitator! Ask groups to clarify their ideas, ask students to share their failures as well as the successful strategies they used, ask more questions to get the students to do more in-depth explaining (Brooks & Brooks, 1999).

13. Yes, give students time to mull over difficult problems. One class period is not always enough to solve a really difficult problem, present it, and discuss it. Leaving a problem ‘up in the air’ piques students’ interest. Laurel remembers the day when “…thirty
minutes passed, no groups had a sure method formulated to solve the problem.” Laurel maintained her position as facilitator, and these students continued working on the problem the following day until they figured out strategies leading to viable solutions.

Given these situations, students often go home talking about the problem to their families or other classmates. How often do students talk about math and show a real interest in it? Not often enough (Burns, 1994). This is a way to make those conversations happen.

How well did you do?

Did you agree with the answers? Remember, these answers are based on specific teachers’ experiences, and do not purport to be the definitive answers or to cover every element of problem solving scenarios. They will however give you a glimpse of what happens when teachers use effective strategies to help their students become critical thinkers and problem solvers.

In self-help tests such as the ones you see in magazines at checkout isles of grocery stores, a score would help you determine your aptitude based a range of correct answers. Instead of telling you that a specific score means you are a novice, an apprentice or an expert, this quiz is designed to make you think about your practice and compare it to these experienced teachers. So how do you think you did?

References


