Cahill’s Conjecture

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Cahill’s conjecture turns into an interesting investigation of mathematics. It was derived after he (a student in class) presented his solution to a problem that he was assigned for homework.

The Problem

\[ \begin{align*}
\triangle ABC & \text{ and } \triangle DBC \text{ are isosceles. } \\
AB &= AC \text{ and } DC = DB. \\
\end{align*} \]

Based upon his observation of the angle measurements in the diagram, he concluded that \( \angle BDC \) and \( \angle BAC \) were supplementary. Thus, he was able to quickly obtain the solution.

The Solution: \( x = 38 \) and \( y = 76 \)

Cahill’s solution worked for the problem that he presented but his method did not work for the following problem. (WHY?)

Similar Problem

On this problem, using his method, Cahill’s solution would be \( x = 28.5 \) and \( y = 57 \).

Question

What relationship must exist between the given angles so that Cahill’s method of solution works?

Cahill’s Conjecture

The class formulated the following statement which was appropriately called Cahill’s Conjecture.

If two isosceles triangles share a common base, \( BC \), and the vertex angle of triangle \( ABC \) is twice one base and of triangle \( DBC \), then the vertex angle of triangle \( DBC \) is twice one base angle of triangle \( ABC \).

Generalization

Once stated, the class generalized and proved that Cahill’s conjecture was true for all cases.
\[ \triangle ABC: \quad a + x + x = 180 \quad \Rightarrow \quad 2x = 180 - a \]
\[ \Rightarrow \quad x = 90 - \frac{1}{2}a \]
\[ \triangle DBC: \quad \frac{1}{2}a + \frac{1}{2}a + y = 180 \quad \Rightarrow \quad a + y = 180 \]
\[ \Rightarrow \quad y = 180 - a \]

Therefore, \( y = 2x \). 
Thus, Cahill’s Conjecture is true.

A student’s method of solution can enhance the learning of every student in class if we, the teachers, are willing to adjust our lesson plan and take the opportunity to investigate the solution with the class.

Try These

To conclude, try the following two problems with your classes. Have the students do the following:

1. Solve each problem.
2. Make a connection between the given number(s) and the solution.
3. Make a conjecture.
4. Write the conjecture in “if, then” form.
5. Prove or disprove the conjecture.

Problem 1:
\( ABCD \) is a square. Point \( M \) is the midpoint of \( CD \). \( AM \) and \( DB \) intersect at point \( Q \). \( QR \perp AD \). \( AB = 8 \). Determine \( QR \).

Problem 2:
\( a \parallel b \). Determine the value of \( x \).

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Front Row (seated) L to R---Ryan Rumph, Alyssa Montalto, Sean Cahill, Leanna Anderson, Liz Awotwi
2nd Row (seated L to R---John Borsellino, Jackie Neustadt, Dalila Camacho, Megan Soger, Emily Kopija
3rd Row (standing) L to R---Jackie Glowinski, Mike Rako, Devon Marino, Samantha Kanak, Melissa Farrell, Kara Schubert, Ashley Saviano
Back Row (standing) L to R---Patrick Baier, Dave Albaugh, Dave Lembas, Garrett Goebel, Dex Jones, Dan Dowjotas, Anthony Harding